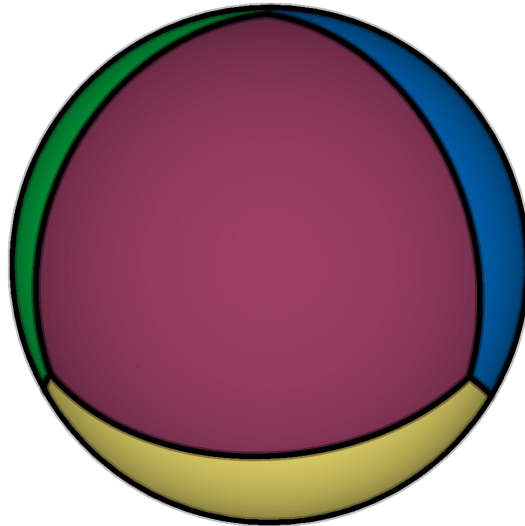

The Complete Homemade Juggling Beanbag Guide

4-Panel Spherical Tetrahedron Chapter




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This is part of a multi-document guide. Hyperlinks with the  icon¹ open other guide documents², if they are saved to the same folder (**CTRL+Click** opens them in a new tab).

Visit my website to download those, and check back occasionally for revisions and corrections. Compare the Last Edited date above on the right with the one on the web page to see if I have submitted changes.

Please contact me with your thoughts! Feedback on this project would be helpful and encouraging. You may also request custom patterns or other help with your project.

If this guide is useful to you, please **consider donating at my website**. I am not monetizing the guide, and I am in need of income.

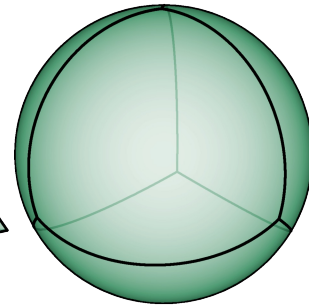
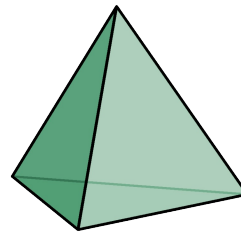
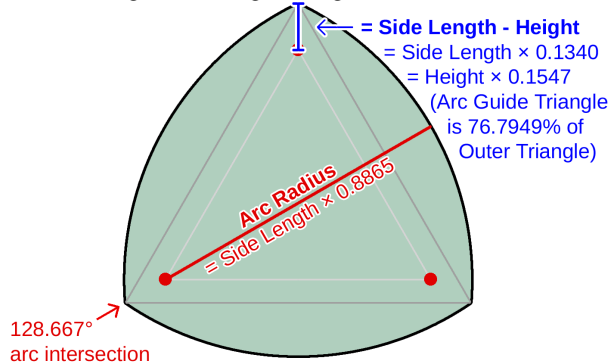
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4-PANEL SPHERICAL TETRAHEDRON INSTRUCTIONS

Panel Triangle Side Length = Bag Circumference \times 0.3381



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Design Notes

This design is about as **easy and simple to make** as the 4-panel Orange Peel Ball, but with a more uniform seam and vertex distribution. It is about as **pyramidal in shape** as the orange peel ball is cubic (only mildly so with fabrics that can stretch), so which design you choose will, apart from your preferred aesthetics, depend on which type of non-spherical shape you prefer the feel of. **Pressing the completed ball against a hot iron** along all the seams will smooth and round the seams and reshape the panels into a more spherical shape. Using a flexible or stretchy fabric, or loosely filling it will also help it feel more spherical. My corduroy bag is surprisingly spherical when I fill it very tightly to stretch out the panels as much as they can. In 2024 I tested the pattern with a fairly stiff faux leather (marine vinyl) and found that it works quite well, at least for those who like a mildly pyramidal feel.

With woven fabrics and filled tightly, this ball design will be a little lopsided due to the fabric's unbalanced direction of stretch. Do not be discouraged and blame yourself for this. (I photographed my corduroy ball at angles that displayed its best shape, and I patted the vinyl ball into a better shape.)

Supplies

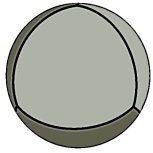
- **For the templates**
 - Cardboard or Template Plastic, Scissors, Glue Stick or Double-Sided Adhesive Tape (to attach the pattern to the template material). **For drawing the pattern by hand:** Paper, Compass, metric Ruler, fine-point Pencil.
- **For the beanbag**
 - Fabric, Needle and durable Thread, Scissors, Fabric Marker or soft Pencil, beanbag Filler, Funnel.
- **For your information**
 - Unless you are experienced with this sort of thing, I recommend that you browse through the **General Information and Techniques** chapter (in the ***01 – Homemade Juggling Beanbag Guide – Index & Supplementary Chapters*** document) before starting. You may find some tips there that will improve your experience and your beanbags.

Printing and Drawing the Pattern

Later in this chapter I provide ready-to-print patterns. (When printing them, be sure to tell the Print Dialog to print only the page(s) you want so you don't print the entire document.) After those are sizing

[formulas](#), [pre-calculated pattern measurements](#), and [instructions](#) for drawing the pattern yourself. Click the hyperlinks or look to the Chapter Index to locate those sections.

Color Arrangements



Other than each panel being a different color, **there is only one balanced color arrangement**: two colors, each on a pair of panels as shown on the left. Since every panel is adjacent to every other, you can choose any two panels for colors A and B.

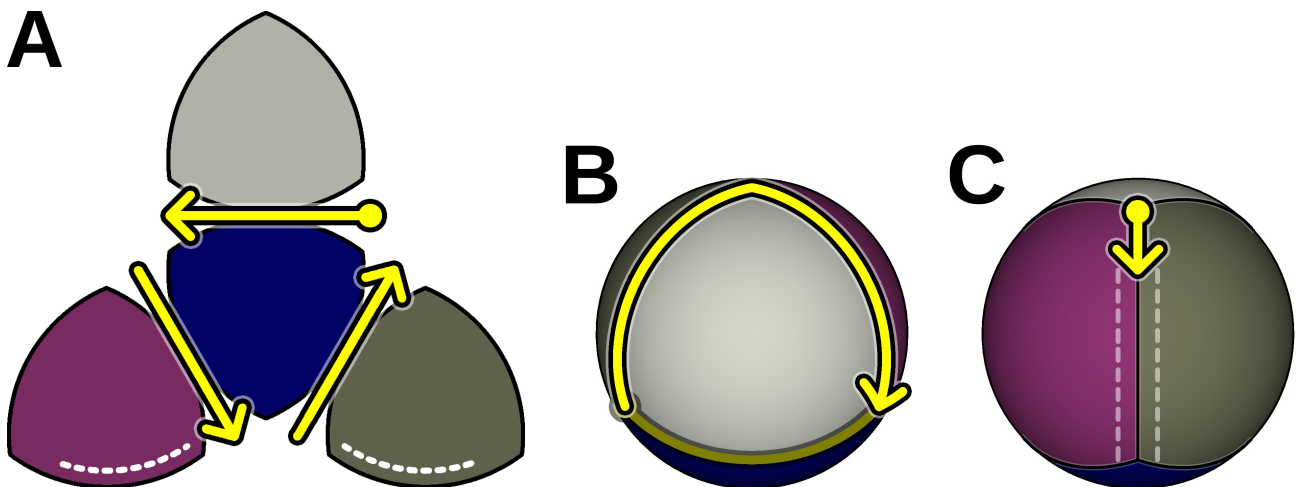
Making the Panels

1. You will need 4 panels, and **you will be tracing the patterns onto the back of the fabric (the side that will be inside the bag)**. If you use a cutting template, first trace that.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well (centering it allows you to align the patterns more easily as you sew, but is not otherwise important).
3. Cut out the panels.

Assembly

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My preferred assembly method is depicted below and uses two or three threads. The method consists of sewing the side panels to the central panel, then sewing up and back down the remaining two sides of the top, gray panel. I begin to sew the final seam with the front stitching pattern (the dashed lines) and then invert the bag and sew the remaining seam from the outside.



Note that in illustration C the ball is still inside out and so the front stitching patterns (the dashed lines) won't actually be visible. I show them just for positional reference.

1. **Illustration A: Lay the panels out as shown** (I prefer to place them front face up).
2. Use the stitching template to **draw partial (or full if you prefer) stitching lines on the fronts** of the two panel edges shown with dashed lines in the diagram. My assembly method will leave these two edges partially unsewn so the bag can be turned out between them. They will then be **sewn from the outside following the front stitching lines**. You will probably not need the entire

seam to be open, which is why I only drew the front pattern on part of it. Be sure to align the template as well as possible with the stitching patterns on the backs.

If you want to **hide the stitching lines within the seam**, sketch them a millimeter or two away from the template (nearer to the panel edges) and stitch slightly inside them (toward the middle of the panels).

I have found it helpful to **add marks along the front stitching lines for each stitch** so that I can more easily keep the exterior stitches even with each other and not get a skewed seam. I space the stitch marks $\frac{1}{8}$ " (3mm) apart. If you **make these marks on your template first**, you can more easily transfer them onto these and future panels.

3. **Illustration A (stitching):** Starting at one of the corners that are not between the front stitching lines (the dot on the top arrow is my suggested starting point), **sew around the central panel, attaching each panel to its edges, with the front faces together** so the bag will be inside out.
4. **Illustration B:** Either tie the thread off or continue it. **Stitch up along two adjacent side panels and then down the other side.** Be sure to stitch along the seams that do *not* have the front stitching lines. Tie off the thread.
5. **Illustration C:** **Begin sewing the final seam with the front stitching lines**, but leave an opening large enough to turn the bag out through it. To **reduce the number of stitches you need to make from the outside**, you can make extra stitches and then loosen them to allow the panels to spread enough to turn the bag out. Then you can pull them tight again from the outside. If you want to do this, or if you want to be able to loosen the last several stitches enough to push a funnel between them, **your final thread will need several inches of extra length.**
6. At this point, **consider ironing the seam allowances flat**; see the [General Information and Techniques](#) chapter under "[Better Seams by Ironing](#)".
7. **Turn the bag right side out through the opening.** A good method for this is to use the back end of a pen or other slender tool to push the fabric through the opening from the opposite side and then either invert the bag around the tool, or remove the tool and work the bag through with your fingers. **Be gentle so as not to pop any stitches.**
8. **Pull out the last stitch so that the thread is on the outside** where you can get to it. Continue sewing the opening closed following the front stitching lines. For help, see the "Stitching Techniques" section of the [General Information and Techniques](#) chapter under "[Backstitch from the exterior Approaches](#)". Fill the bag at some point during this final sewing with a funnel. I find it helpful to **put some filler in first to prevent the bag from collapsing** while I sew, and to hold the seam allowances in place and give me something to push the needle against. You can **sew the entire opening closed before fully filling the bag**, which prevents the filler from spilling back out while you sew. Just loosen the last several stitches enough to push the funnel between them, or at least to push some filler in with your fingers. Then re-tighten the stitches (see "[Tips on finishing the bag](#)").
9. **To reshape the finished bag into a better sphere and reduce angularity and lumpiness at the seams, [press it against a hot iron along all the seams](#)** (be sure to use an appropriate heat setting).

Ready-to-Print Patterns

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The pattern pages are 8.27"×11" (210mm×279mm) to fit both "Letter" and "A4" sizes. **Make sure the print is not being scaled to fit the printer margins** (select Default/None scaling/Actual size/Ignore printer margins). To verify correct sizing, **compare the centimeter grid to a ruler** and adjust the next print if necessary. (Note that PDF viewers and printers can both contribute to slight size inaccuracy.)

On the following pages are patterns for beanbag diameters from 2" – 3" in $\frac{1}{4}$ " increments, and a 6" pattern for scaling to larger sizes. The patterns are reduced by 6.6% from the mathematical calculation to account for the inflation in size I observed in my corduroy bag. **If you use a completely non-stretch fabric, I recommend enlarging the pattern to about 103% to get the intended ball size.**

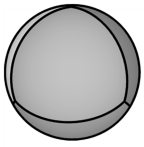
To make the templates, I recommend cutting out the portions of the paper with the patterns you want and using glue or double-sided tape to attach them to your template material, and then cutting along the patterns.

The cutting patterns have 4mm, 6mm, and 8mm allowances so you can choose the amount that works best for your fabric and preference (the lighter, 6mm cutting pattern is a hair under $\frac{1}{4}$ "). Remember that the cutting patterns have slightly different curve radius to panel size proportions from the stitching patterns (they are parallel, not proportional), so **you should not use them as stitching patterns.**

To produce other pattern sizes or correct an incorrectly sized beanbag, adjust the size scaling in the print dialog. For example, to reduce my 2.5" pattern to the 2.3" size recommended by the Juggling Store for small hands and numbers juggling, divide 2.3 by 2.5, multiply the result by 100, and that is your scale (92% in this case). If your beanbag ends up not being the expected size, see the [General Information and Techniques](#) chapter under "[Adjusting/Scaling a Pattern to Produce an Accurate Ball Size](#)" for help with correcting it.

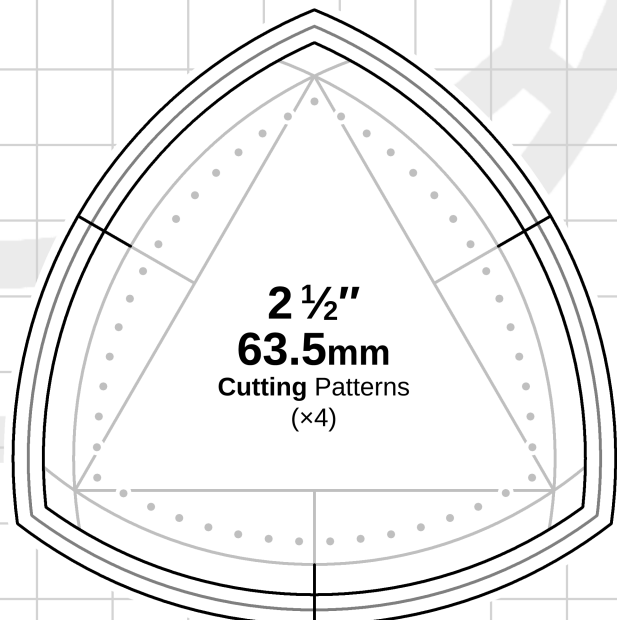
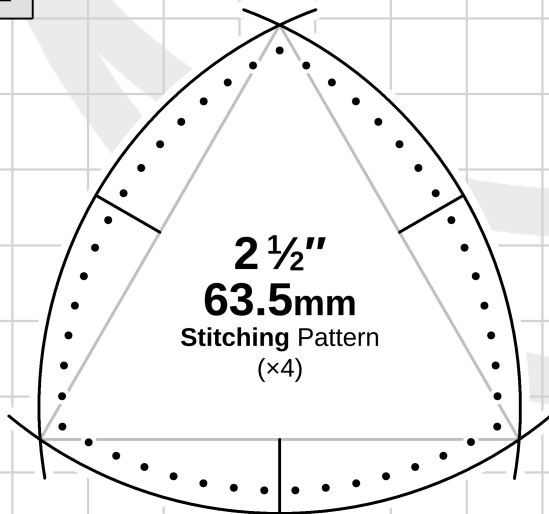
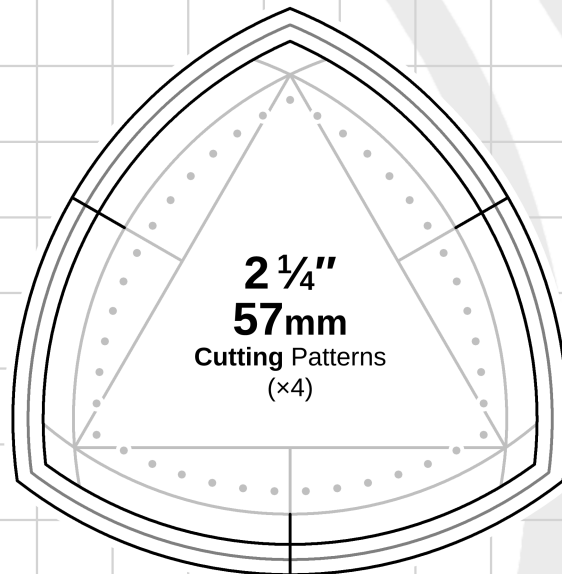
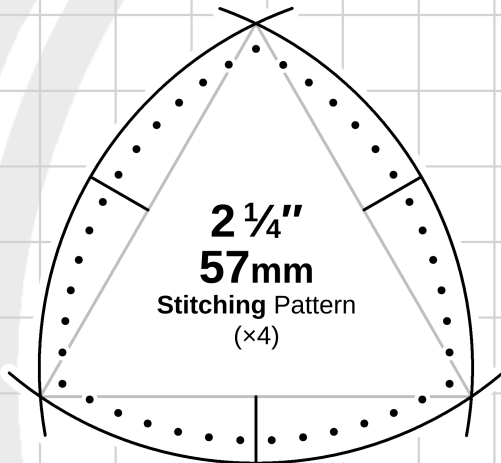
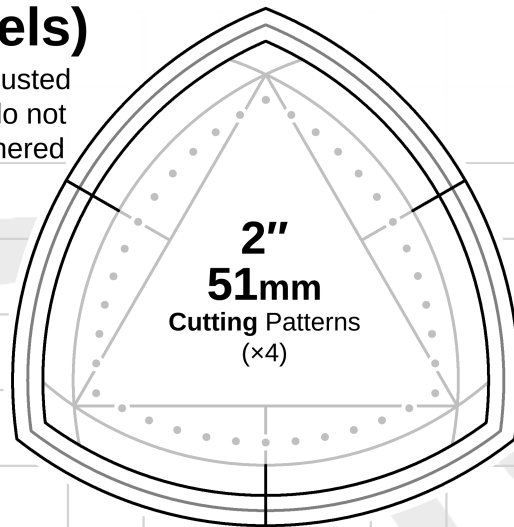
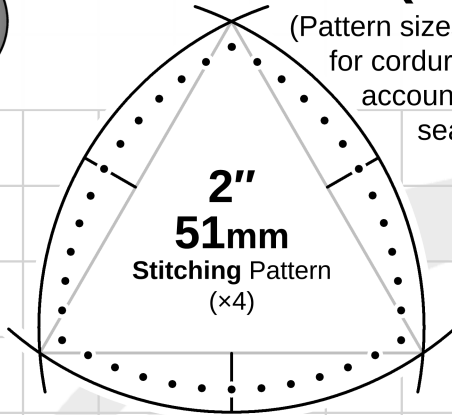
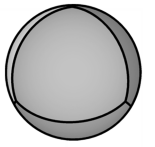
The table below provides the scaling for the $\frac{1}{8}$ " increments between my $\frac{1}{4}$ " sizes. The centimeter grid can be used to verify correct scaling.

Target Diameter	Print this pattern size	At this scale
1 $\frac{3}{4}$ " (44.5mm)	2"	87.5%
1 $\frac{7}{8}$ " (47.6mm)	2"	93.8%
2 $\frac{1}{8}$ " (54.0mm)	2 $\frac{1}{4}$ "	94.4%
2 $\frac{3}{8}$ " (60.3mm)	2 $\frac{1}{2}$ "	95%
2 $\frac{5}{8}$ " (66.7mm)	2 $\frac{3}{4}$ "	95.4%
2 $\frac{7}{8}$ " (73.0mm)	3"	95.8%

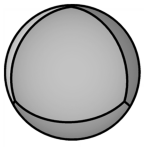


Tetrahedron (4 Panels)

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)

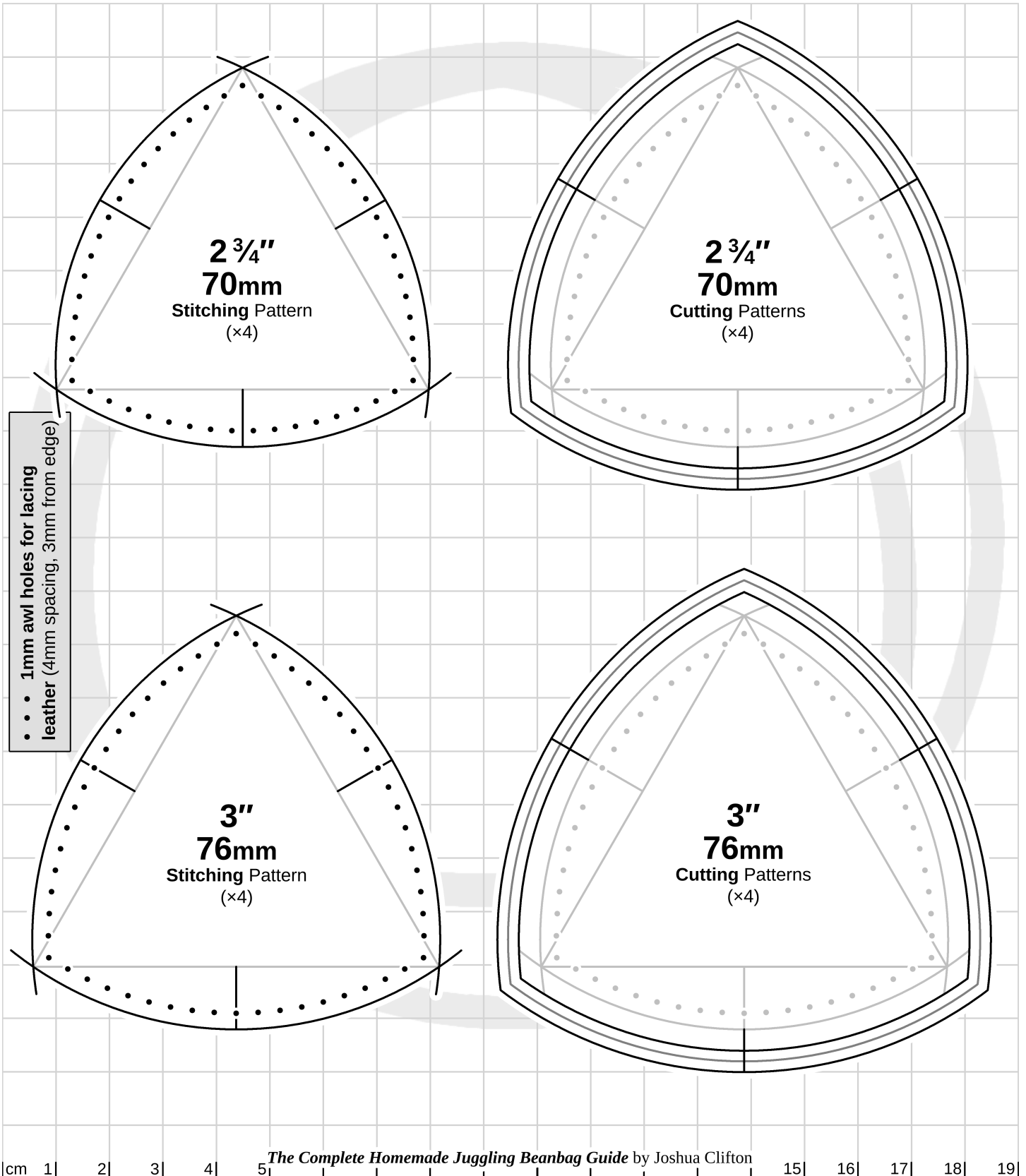
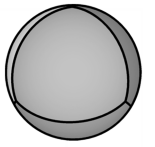


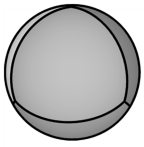
• • • 1mm awl holes for lacing leather (4mm spacing, 3mm from edge)



Tetrahedron (4 Panels)

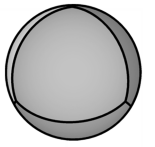
(Pattern sizes are adjusted for corduroy and do not account for gathered seams)



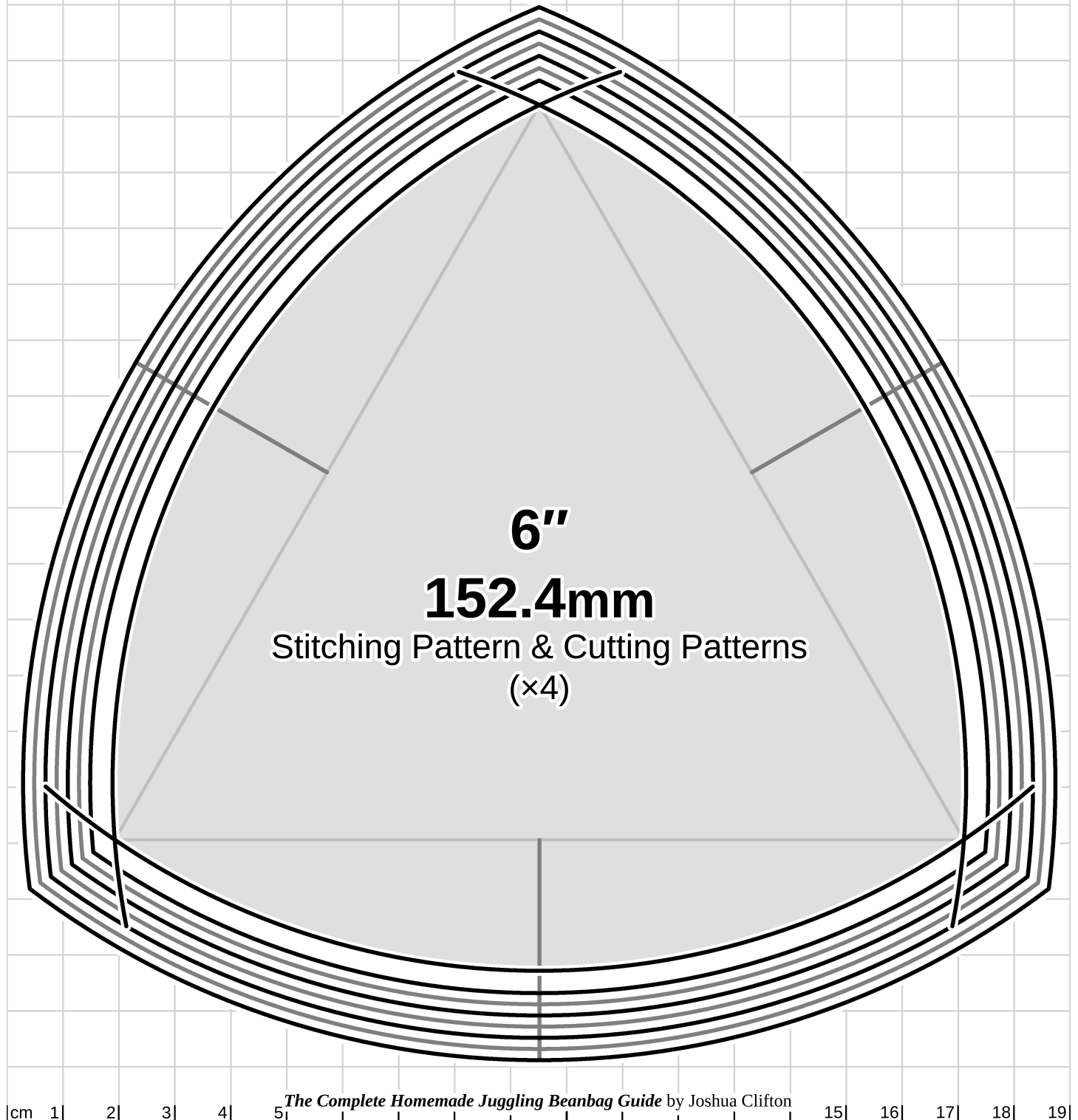


Tetrahedron (4 Panels)

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)



Extra large and versatile pattern for scaling to larger sizes in the Print Dialog. Print twice if you want both a stitching template and a cutting template. The inner pattern (filled with gray) is the stitching pattern. Each dark pattern outside of that marks a 4mm seam allowance interval (at 100% scaling). Use those or the lighter, half-intervals between them to cut out the amount of allowance you want for the cutting template.



Sizing Formulas for Drawing the Pattern

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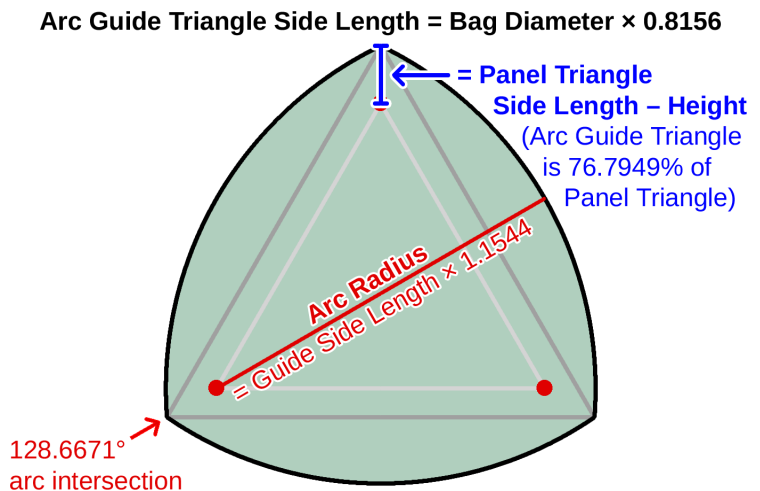
The next section has a table of pre-calculated pattern measurements for all $\frac{1}{8}$ " diameter increments from $1\frac{3}{4}$ " – 3". Following that are the drawing instructions. If you do not need to create a custom size, skip to that. I provide [printable measuring tapes](#) at the end of the **General Information and Techniques** chapter in case you care to measure your beanbags. The “Mathematics” section has explanations of all the formulas and ratios, and expresses their values in higher precision.

Design summary

The panel shape is formed by creating the three corners of an equilateral guide triangle to use as circle centers/compass points for arcs that form a circular triangle around the guide triangle.

The circumference of the bag could be measured by the height of two panels and the length of the arc forming the edge, but I found that in practice this produces a bag that is slightly too small compared to the orange peel ball, cube, and octahedron when made with the thin, stiff fabric I used for the test

bags. To make this design’s sizing consistent with the others I based the panel size calculation on the actual bag circumference. The panel height is therefore 34.5% of the desired circumference. From there I can calculate the other elements of the panel design.



Adjusting for the influence of fabric types on beanbag size

The bag I made with thick corduroy was **5.01 – 8.23%** larger than the mathematical prediction depending on whether I filled it loosely or over-filled it. I target halfway between the min and max inflations when sizing my patterns, which is **6.6%**. This makes my adjustment factor **1.066**. The bag I made with a fairly thin, stiff, tightly-woven, non-stretch fabric was 2.42% - 5.11% larger, but that was just for analyzing the shape characteristics of the bag. Corduroy is a better choice for juggling, so I will use that adjustment factor.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under “[Adjusting/Scaling a Pattern to Produce an Accurate Ball Size](#)”.

Sizing formulas

Below are the formulas to calculate the pattern construction elements (*Diameter* and *Circumference* refer to your target ball size). The value in orange is the adjustment factor. **Don’t forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

- **Guide Triangle Side Length** = $Diameter \times 0.8156 \div 1.066$
= $Circumference \times 0.2596 \div 1.066$
- **Guide Triangle Circumradius** = $Diameter \times 0.4709 \div 1.066$
= $Circumference \times 0.1499 \div 1.066$
- **Arc (circle) Radius** = $Side\ Length \times 1.1544$
= $Radius \times 1.9995$
- **Resulting Panel Height** = $Diameter \times 1.0838 \div 1.066$
= $Circumference \times 0.3450 \div 1.066$ (my chosen panel sizing factor)

Forming the panel shape given an equilateral starting triangle

If you want to convert an equilateral triangle into the panel shape by adding curved sides to it, here are the calculations (s = starting triangle Side length, h = triangle Height, r_t = triangle Radius):

- **Guide Triangle Circumradius** = $r_t - (s - h) \approx 0.7679r_t$ (that is, the circle centers are located inside each of the triangle's corners by the distance of $s - h$, for which non-arbitrary reason I chose this curve to try)
- **Guide Triangle Side Length** = $0.7679s$
- **Arc (circle) Radius** = $0.8865s$

Arc (edge) length for spacing awl holes or stitch marks

To [calculate the length of the curved edges](#), use the following formula, plugging in the Arc Radius you calculated above. (34.3333° is the angle between the arcs, or rather tangents thereof, and the edge of the triangle they span.) If you are working with Radians, omit the $\pi/180$.

$$(Arc\ Radius)(2)(34.3333)\left(\frac{\pi}{180}\right) \approx Arc\ Radius \times 1.1985$$

Table of Pre-Calculated Pattern Measurements

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The table below has stitching pattern measurements for each $\frac{1}{8}$ " diameter increment from $1\frac{3}{4}$ " to 3". The values in the **Adjusted** columns account for my 1.066 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using a thick fabric like denim or corduroy.

To draw the cutting pattern, use the same Guide Triangle, but increase the Arc Radius by the desired seam allowance (I use 8mm) and center the three new arcs at the same three points. The cutting pattern will be larger than, but parallel to, the stitching pattern.

Finished Diameter	Guide Triangle Side Length (mm)		Guide Triangle Circumradius (mm)		Arc Radius (mm)		Panel Height (mm) (for double-checking)	
	Base	Adjusted	Base	Adjusted	Base	Adjusted	Base	Adjusted
1¾" (44.5mm)	36.255	34.010	20.932	19.636	41.852	39.261	48.177	45.194
1⅞" (47.6mm)	38.845	36.440	22.427	21.038	44.842	42.065	51.618	48.422

Finished Diameter	Guide Triangle Side Length (mm)		Guide Triangle Circumradius (mm)		Arc Radius (mm)		Panel Height (mm) (for double-checking)	
	Base	Adjusted	Base	Adjusted	Base	Adjusted	Base	Adjusted
2" (50.8mm)	41.434	38.869	23.922	22.441	47.831	44.870	55.060	51.651
2½" (54.0mm)	44.024	41.298	25.417	23.844	50.820	47.674	58.501	54.879
2¾" (57.2mm)	46.613	43.727	26.912	25.246	53.810	50.478	61.942	58.107
2⅝" (60.3mm)	49.203	46.157	28.407	26.649	56.799	53.283	65.383	61.335
2½" (63.5mm)	51.793	48.586	29.903	28.051	59.789	56.087	68.824	64.563
2⅝" (66.7mm)	54.382	51.015	31.398	29.454	62.778	58.891	72.266	67.791
2¾" (69.9mm)	56.972	53.445	32.893	30.856	65.768	61.696	75.707	71.020
2⅞" (73.0mm)	59.562	55.874	34.388	32.259	68.757	64.500	79.148	74.248
3" (76.2mm)	62.151	58.303	35.883	33.661	71.747	67.304	82.589	77.476

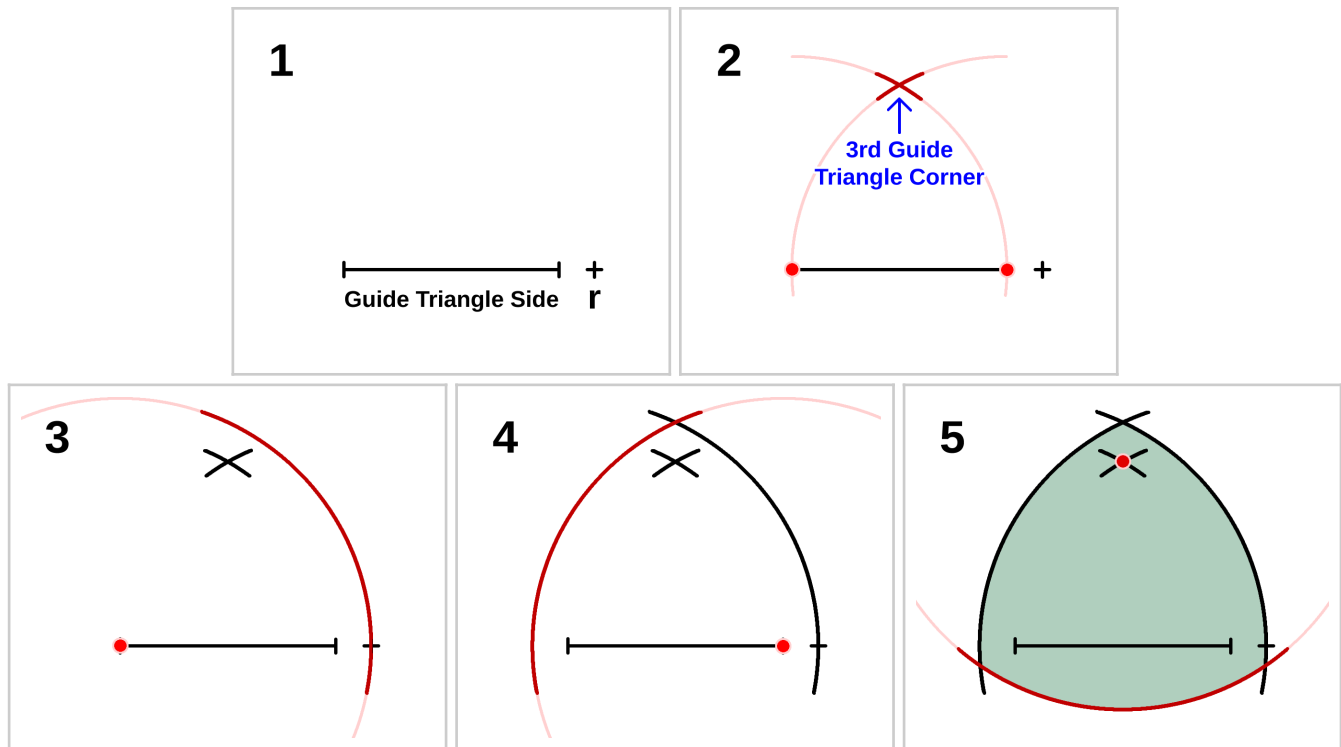
The drawing instructions begin on the next page.

How to Draw the Panel Shape

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The panel shape is based on a regular (equilateral) triangle but has circular sides to produce a more spherical bag. I draw this shape by constructing an equilateral guide triangle (actually just enough of it to get the three corners), setting the circle/compass radius to 115.4385% of the length of the triangle's sides, and then, using the three corners as compass points, drawing the circular triangle around the guide triangle.

The illustrations below are oriented toward drawing the shape by hand and their numbers correspond to the step numbers in the manual directions. After the manual directions are SketchUp directions. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.



Illustrations for the manual directions. The numbers correspond to the step numbers.

Manual directions

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a horizontal line the length of **Guide Triangle Side Length** and mark each end of it. This is the base of an imaginary equilateral guide triangle. Mark a point (labeled *r* in Illustration 1) the distance of **Arc Radius** from the beginning of the line (it will lie beyond the end of the line) which will be used to extend the compass to the correct radius.
2. Before using that radius measurement, set the compass radius to the length of the line and draw an arc from each end of the line to form an X above its center, marking the third corner of the Guide Triangle.
3. Now use the **Arc Radius** mark (the point labeled *r* in Illustration 1) to expand the compass to that radius, and then draw an arc that extends from a little below the line up to directly above the center as in Illustration 3.

4. Keeping the compass radius unchanged, place the compass on the other end of the line and draw another arc like the previous one.
5. Place the compass on the intersection of the first two arcs (the 3rd Guide Triangle Corner) and draw a third arc below the Guide Triangle Side, joining the previous two arcs and completing the panel shape. The circular triangle's height (corner to middle of opposite arc) should equal the specified **Panel Height**.
6. To draw a cutting pattern, draw everything the same but increase the Arc Radius by the desired seam allowance (I use 8mm) and then draw the last three arcs from the same three points using that new radius.

SketchUp directions

(The terms in bold refer to columns in the pattern measurement table above.)

1. Use the polygon tool (in the Shapes tool drop-down, or in Draw menu -> Shapes) set to 3 sides and draw a triangle with radius = **Guide Triangle Circumradius**, which will result in a triangle with sides of length **Guide Triangle Side Length**.
2. Draw circles with the specified **Arc Radius** centered on the three corners of the guide triangle. The intersection of the circles forms the circular triangle panel shape whose height (corner to middle of opposite arc) should equal the specified **Panel Height**.
3. To draw a cutting pattern, draw the same guide triangle but increase the circle radii by the desired seam allowance (I use 8mm).

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Mathematics Behind the Relationship Between the Pattern Parameters and the Ball Size

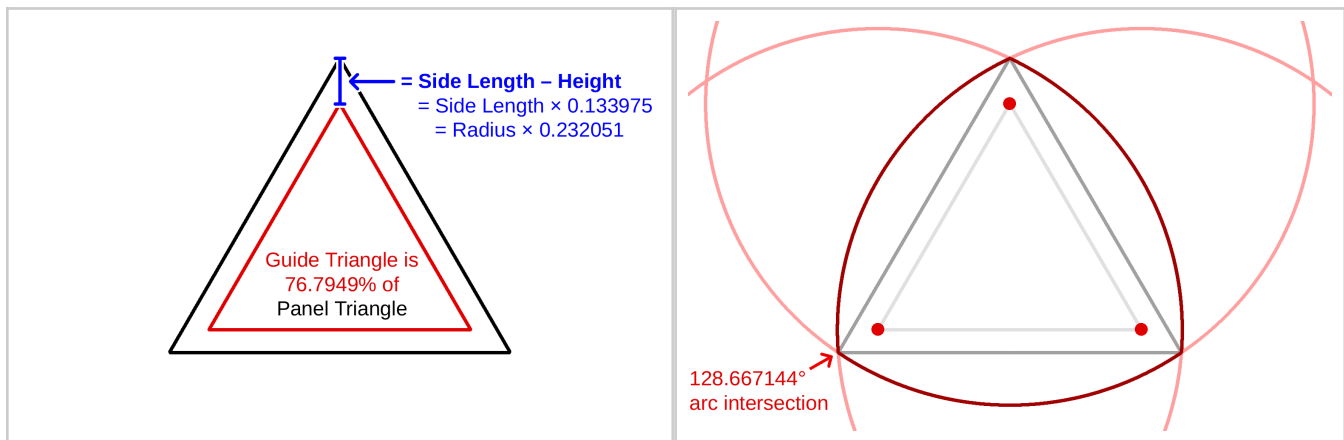
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This section describes the math involved in drawing patterns to produce specified beanbag sizes, and creating the pattern sizing formulas. (The numbers in tiny, right-justified typeface are my computer calculator's unrounded values which I display rounded to six places for brevity.)

I formed this curve by locating the circle centers inward from an equilateral triangle's corners by the distance of *Side Length – Height*. Another way of stating this is that the Guide Triangle's circumradius (center to corner) is shorter by that much than that of the Panel Triangle. This curve (its tangents, technically) produces a 128.667° angle at the corner for a vertex sum of 386° . There is a reason why I chose that curve of all the possible curves. I discuss that in the “How I Developed This Design” section. In short, it is a non-arbitrary choice that works better than curves that are slightly steeper and slightly shallower than this one, including the curve that forms a 120° angle at the intersections (a Reuleaux triangle), which is the mathematically correct angle because it produces a vertex sum of 360° .

Because of the shape of the tetrahedron, the bag circumference is not as predictable by math as the other designs. Two panel heights plus one panel edge should work, but I found that it made the bag too small compared to other designs. In the end I chose **Circumference** $\times 0.345$ as the definition of the panel height, which I calculated from a bag I made from a thin, non-stretch fabric and measured. I use that ratio to calculate the distance between circle centers for a target bag size, and I calculate the rest of the panel design dimensions from that.

Following are diagrams of the panel shape derivation and relevant ratios and formulas.



The circumradius of the panel triangle, assuming edge lengths of 1, is $1/\sqrt{3} = 0.577350$, and the circumradius of the guide triangle is reduced by *Side Length – Height* or $1 - \sqrt{3}/2 = 0.133975$. So the guide triangle's circumradius is 0.443376. Thus:

Ratio of Panel Triangle:Guide Triangle $\approx 1:0.767949$

0.7679491043122706472053868654

Distance between Guide Triangle Corner and Panel (outer) Triangle Corner

= Side Length – Height

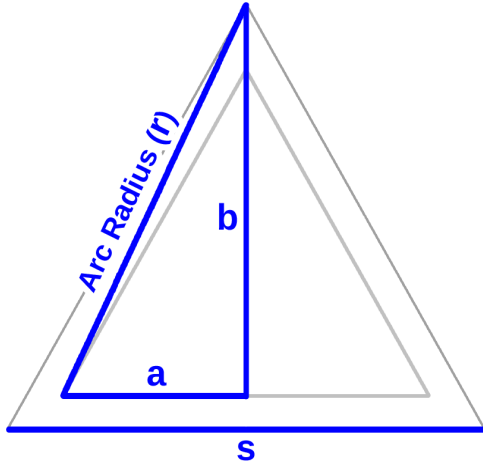
\approx Side Length $\times 0.133975$

0.13397456621556135329527682624706

\approx Radius $\times 0.232051$

0.2320506798872205274462432595

To calculate the arc radius in terms of the panel triangle's side length, s , I need to solve a right triangle as illustrated below. The arc radius is the hypotenuse of the right triangle.



The panel triangle's circumradius, r_t , is the distance from the center to the corner, or $\frac{2}{3}$ of the height, and the apothem, p , is the distance from the center to the edge, or $\frac{1}{3}$ of the height. Side b in the illustration is equal to the radius plus 0.767949 of the apothem.

$$a \approx 0.5(0.767949)s \approx 0.383975s$$

$$b \approx r_t + (0.767949)p \approx 0.577350s + (0.767949)(0.288675s) \approx 0.799038s$$

$$r = \sqrt{a^2 + b^2} \approx \mathbf{0.886509s}$$

I will use the following equilateral triangle formulas³ to calculate the ratios that follow them (s = side length):

$$\text{Triangle Height/Altitude, } h = \frac{\sqrt{3}}{2}s \approx 0.866025s$$

$$\text{Circumradius (center to corner)} = \frac{s}{\sqrt{3}} = \frac{2}{3}h \approx 0.577350s$$

$$\text{Apothem/inradius (center to edge)} = \frac{\sqrt{3}}{6}s = \frac{R}{2} = \frac{h}{3} \approx 0.288675s$$

Arc Radius, $r \approx 0.886509s$

- In terms of the Guide Triangle Side, s_g : $\frac{0.886509s}{0.767949} \approx \mathbf{1.154385s_g}$

- In terms of the Guide Triangle Radius, r_g : $\frac{1.154385s_g}{0.577350} \approx \mathbf{1.999454r_g}$

Now that I have defined the arc radius, I can calculate the sagitta, which is the height of the apex of the curve above the triangle edge. With that I can calculate the height of the panel. The formula for the sagitta is the following:

$$\text{Sagitta} = r - \sqrt{r^2 - (0.5s)^2} \approx 0.886509s - \sqrt{0.785898s^2 - 0.25s^2} \approx \mathbf{0.154458s}$$

$$\text{Panel Height} \approx 0.866025s + 0.154458s \approx \mathbf{1.020484s}$$

$$\approx (1.020483s)(0.767949) \approx \mathbf{1.328843s_g}$$

Since, by definition,

$$\text{Panel Height, } h_p, = \text{Bag Circumference} \times 0.345$$

and I have expressed the Panel Height in terms of the Guide Triangle, I can now express the **Guide Triangle's Side Length (or distance between circle centers), s_g** , and **Circumradius, r_g** , in terms of

³ Thanks to Wikipedia: https://en.wikipedia.org/wiki/Equilateral_triangle

the Bag Circumference, C and Diameter, D , so that the pattern can be sized according to the desired bag size.

$$s_g \approx \left(\frac{h_p}{1.328843} \right) (0.345 C) \approx \mathbf{0.259624C}$$

0.259624C000000113203094334807620

$$\approx (0.259624C)\pi \approx \mathbf{0.815634D}$$

0.815634D000000113203094334807620

$$r_g \approx (0.259624C)(0.577350) \approx \mathbf{0.149894C}$$

0.149894C000000113203094334807620

$$\approx (0.815634D)(0.577350) \approx \mathbf{0.470907D}$$

0.470907D000000113203094334807620

Cutting pattern adjustment

To make a cutting pattern, simply increase the arc radius by the desired seam allowance. The guide triangle remains the same.

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How I Developed This Design

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Photos from <http://www.renegadejuggling.com/4-Panel-Suede-Ball-p116.html>

Original pattern design

The Renegade suede tetrahedral bags (shown above) looked pretty good and I became interested in designing a tetrahedral bag back during the writing of the first edition of this guide. Peter Billam’s website (from which I got the inspiration for my spherical cube and spherical octahedron designs – see <http://web.archive.org/web/20231105094249/https://pjb.com.au/jug/leatherballs.html>) also has a rounded tetrahedron pattern.

Through measurement and experimentation (and, in June, 2020, confirmed by using an overlay of a SketchUp-created pattern in Photoshop) I determined that the panel shape Billam used for this pattern is a Reuleaux triangle. I tried making a Reuleaux tetrahedral beanbag and it was so non-spherical that I did not deem it worth including formal instructions for in this guide. As Billam puts it, “it has a shape a bit like an egg with four ends.” I love that description. It was a cool-looking beanbag, but not as easy to catch as a spherical bag.

I later decided that a logical way to design the curve was to use the circular arc that would be used for a 3-panel orange peel ball. That worked fairly well and is the version I defined in the first edition of this guide.

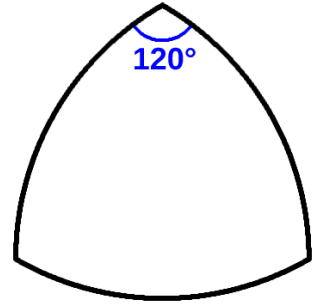
When I was prompted to redesign my octahedron’s panels by a Reddit contributor’s remark that their curves were too steep, I rethought my cube and tetrahedron as well, since they were based on the same curves (all used circular arcs derived from the orange peel ball concept). I experimented with other curves and found a shallower one that worked better. Following is how I arrived at it.

Discovering the vertex-angle-sum design principle

In 2013 or so I had read an article on spherical geometry⁴ which explained that for polygons to form a sphere, the sum of the angles meeting at each vertex must be 360° . I later learned the mathematics to calculate the radius of the arc that will form a specified angle to a polygon edge. The spherical geometry article gave me a starting point of experimentation, and the edge arc radius formula I created enabled me to design new experimental curves for my panels.

⁴ http://euler.slu.edu/escher/index.php/Spherical_Geometry

In the case of the tetrahedron, which has three corners at each vertex, the corners must be 120° , which is a Reuleaux triangle. Peter Billam was evidently aware of this principle. Not only did he use a Reuleaux triangle for his tetrahedron, but all of his other designs' panel bulges, it turns out, create angles at the corners such that the sum of angles in the solids' vertices are 360° . And for all but the tetrahedron he uses simple, angled bulges. I guessed at the derivation of his bulge angles, and then, during the writing of the second edition guide, confirmed it by creating shapes with those angles in SketchUp and overlaying them on enlarged views of Billam's shapes.



Since the Reuleaux triangle did not work well, I experimented with steeper curves, but stayed under the orange peel ball curve. For a full explanation of the mathematics of designing arcs that meet at specific angles, read the ["Curved-Edge Faces" section](#) of Chapter 5. I also discuss ways to deal with the fact that a circular arc that forms the correct angle does not necessarily bulge outward enough to form a good bag shape, and how to recognize the shape characteristics of a beanbag with too shallow or too steep a curvature.

New edge-arc experiments for the Second Edition

My old orange peel curve produced an arc intersection angle of about 134.8° . This caused the vertices of the bag to have a tendency to sink inward a bit. I thought this was a benefit, but it was a little too much. I now think it makes them feel too flat and the edges bulge too prominently.

Marylís Ramos made a PDF set of juggling bag patterns⁵, and it was her octahedron pattern that the Reddit contributor recommended instead of mine. I had found her patterns during my research on the first edition of this guide, but I did not know how she derived her panel shapes. I contacted her in 2016 but got no reply to my question about her design method. She has the most professional and precise-looking patterns of all that I have encountered, so I used her patterns as a reference and basis of comparison.

Ramos' tetrahedron pattern uses a curve that produces a 124° angle (32° between the arc and the triangle side – the tangent-chord angle). I determined this by overlaying SketchUp models on a magnified view of her pattern in Photoshop. I made a bag using that panel shape and it was better than the Reuleaux bag, but it still had a pyramidal shape and somewhat prominent vertices.

I then tried a curve that put the circle centers inward from the triangle's corners the distance of the triangle's side length minus its height (rather than arbitrarily picking a curve). This produces a 34.334° tangent-chord angle, or 128.667° total, and made a much better shape.

Just to be sure, I tried a slightly steeper curve that put the circle centers inward by one third the distance of the triangle's radius. This produces a 36.587° tangent-chord angle, or 133.174° total, and was a little too steep.

It was only during writing this documentation and drawing the diagrams that I realized that the orange peel curve I drew to indicate the upper limit of the curvature was incorrect, being much too steep. The 133.174° curve was significantly shallower than that one. When I redrew the orange peel curve to provide an illustrated comparison, I drew it correctly and found that it was only slightly steeper than this

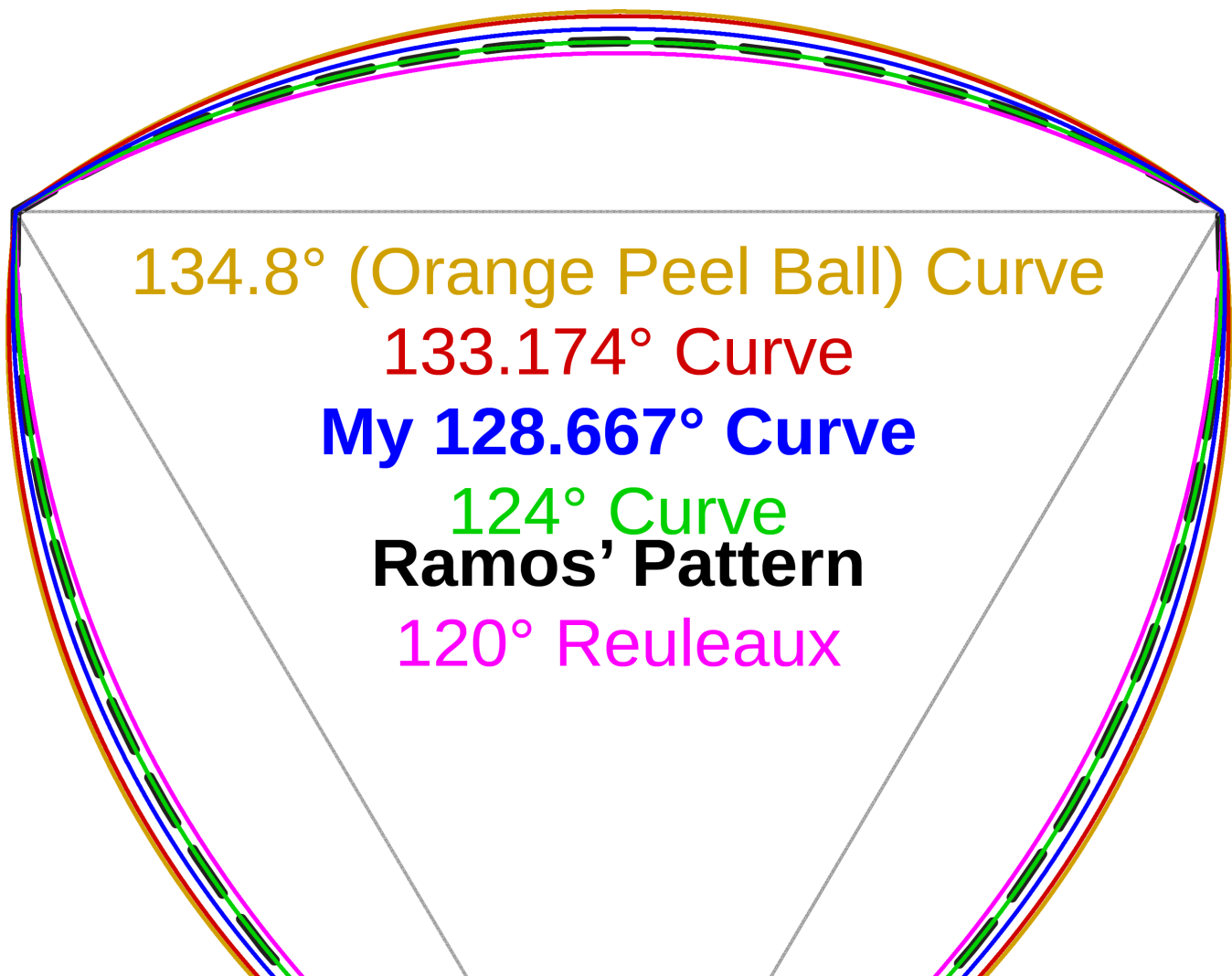
⁵ "Sewing Patterns for Jugglers" [Orange Segment Series](#) and [Polyhedra Series](#).

curve, with an angle only 1.626° greater. Still, it confirmed that this curve was too steep, and, though I didn't at the time think that such a small difference would be noticeable, I later learned through experimenting with small angle adjustments in the cube and octahedron that it actually might have been.

The 128.667° angle seems to be the perfect balance, making the vertices pleasantly rounded without being too flat, and the seams having a curvature that produces a good profile. If they bulged much more, I think their curvature would be too tight for the size of the beanbag.

I made one bag with my design testing fabric (thin, moderately stiff, tightly woven, non-stretch) and one with a thick, soft corduroy. Both feel very good with perfectly balanced vertices and edges. My corduroy bag feels surprisingly spherical for such a non-spherical polyhedron. The one I made with the design testing fabric is very angular, but that is unavoidable with a fabric like that. The seam curvature is not the problem there, but the size of the panel faces, requiring them to balloon out and distort far beyond what that fabric is capable of to produce a spherical shape.

The illustration below is an overlay of all the curves over Marylis Ramos' pattern, which is the black dashed pattern. Coincidentally, my three Second Edition curves (red, blue, and green) are almost exactly evenly spaced, with just over 4.5° difference in the corner angle produced by each successive curve. It is surprising how much difference a small change in the curve can make in the shape and feel of the beanbag.



Testing the pattern with felt and leather

In mid-2021, several months after publishing my second edition guide, I made felt balls with my 2 – 12-panel designs just to make sure my patterns would produce a good shape even with a light, moderately stretchy fabric. They all turned out fine. The tetrahedron, whether moderately filled or tightly filled, exhibited only minor flatness at the vertices, which I think is a benefit because, with a face opposite each vertex, the only way the ball can approximate a sphere and feel symmetrical is by having vertices that are flattened out enough to have a similar shape to the faces. The seam curvature seemed about optimal. I was satisfied with it.

In January, 2024 I found a [Reddit post by Idkmyname2079048](#) displaying a set of beautiful leather, tetrahedral juggling balls that she made for her husband. I had never thought that someone would actually want to use this design to make a leather ball (if a better design was available), considering what a poor sphere it produces with a material that does not stretch much. But it seems from the comments that some people like the pyramidal shape.

So I decided I ought to test my pattern with marine vinyl, which is similar to a moderately stiff leather, and make sure it works. The experiment demonstrated that **my pattern is perfectly suitable for use with leather**.

I have two types of marine vinyl. One, a new type I got recently from Hobby Lobby, has about the same stretch as the leather in the photo appears to have, while the other, from Jo-Ann Fabrics, is stiffer and almost non-stretch. The latter is what I had used for my 2-panel baseball-style beanbag and I experimented first with that.

The stiffer vinyl, I thought, would probably make a better test case. I have had some concern that my pattern's over-wide corners (producing a vertex sum of 386° instead of the mathematically correct 360°) might not fit together well with something that does not distort easily ([see my illustration near the end of the chapter](#)), and might need a Bézier curve modification to correct the corner angle but retain the apex height. The stiff material would best show if the corners produced a good vertex shape.

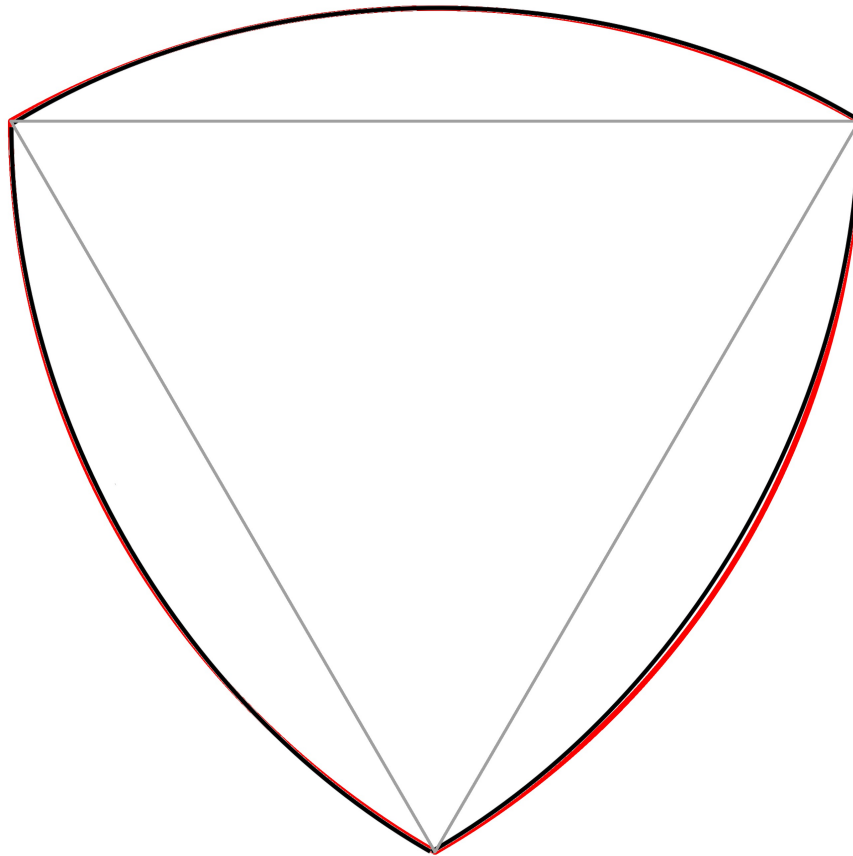
The pattern for the leather balls was by Leathercraft Rebels, [purchased through their shop on Etsy under the name Dogiverse](#). I am always on the lookout for other patterns and beanbag making instructions from which to learn new information and ideas to improve my own, so I bought the PDF pattern and instruction booklet for \$3.95 (half off). The booklet also includes a 4-panel orange peel ball pattern, and the patterns have marks for awl holes. Only one pattern size, 65mm/2.56" ball diameter, is provided.



Leather balls and photo by [Idkmyname2079048 on Reddit](#)

The booklet not only made me aware of the “A4” paper size that Europeans use, which is narrower than the US’s “Letter” size, prompting me to resize my printable pages so that they fit within both paper sizes, but its printing notes caused me to decide to improve my own printing notes in each printable section and move them to the top of the page to make them more prominent. (Those notes are in the Ready-to-Print Patterns section, and are shaded in blue. They used to be at the bottom of the page and had less relevant information.)

As for the pattern, it, like Peter Billam’s, is a Reuleaux triangle (120° corners), but is poorly drawn (see the comparison below). Both the tetrahedron pattern and the orange peel pattern are asymmetrical, proven by flipping/rotating them and overlaying them on the original, but the orange peel pattern is only slightly off. The tetrahedron pattern has one edge arc that is significantly shallower than the others.



Leathercraft Rebels’ pattern in black (awl holes and logo omitted) compared to a Reuleaux triangle in red. The corners and edges of LR’s pattern do not quite align with the Reuleaux triangle, and one edge has a shallower curvature than the other two.

I used my 3” pattern with edge-to-edge, baseball stitch construction, and filled the ball very tightly. It turned out as I hoped, which is about as well as a tetrahedral ball can be expected to with a material like this. The panel corners fit together perfectly and the resulting vertices have a good shape, being neither pointed nor puckered inward, and the seams do not “shrug their shoulders” too high around the vertices, which is a characteristic that can result from edge arcs that are too steep.

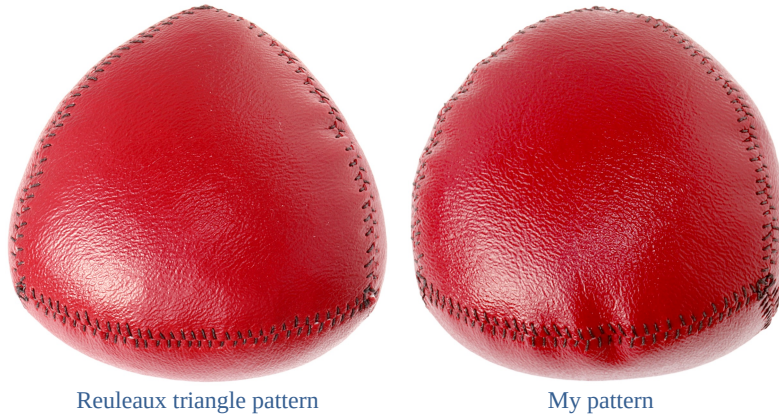
The ball reaffirms that this pattern is very good and is in no need of adjustment (though I have little doubt that some small improvement is possible since my curve angle increments were rather large compared to my other designs).

The seams exhibit some dimpling due to their steep curvature around the ball and the tension put on them by the bulging faces, and I was concerned that the Reuleaux pattern might work better in this

respect. So I drew a Reuleaux pattern of the same size and made a ball with that. The seam dimples on that ball were still present, but were sparser and subtler.

While both balls are very obviously pyramidal both by sight and by feel, the shape of the Reuleaux ball is decidedly inferior to the one made with my pattern. Its vertices are sharp and prominent and feel like corners, whereas mine have practically no prominence at all and are pleasantly rounded. My steeper seam curvature also makes the ball much plumper and more nearly spherical, and the seams conform much better to an overall circular profile. The ball still has a pyramidal feel (a benefit to those who like that) due to the stiff, nearly non-stretch marine vinyl, but is also near enough to a sphere to be fairly easy to catch. It is easier to catch and grip than the Reuleaux ball.

Vertex and Plumpness Comparison



Reuleaux triangle pattern

My pattern

Seam Profile Comparison



Reuleaux triangle pattern

My pattern

I then made a ball using my pattern with the stretchier, more flexible marine vinyl. That ball is as much rounder, plumper, and blunter at the vertices than the one made with the stiffer vinyl, as that one is than the Reuleaux version. It still looks and feels unmistakably pyramidal, but only mildly so. Perhaps not enough for someone who loves a pyramidal ball, but my goal is to produce spheres. Seam dimpling on the new ball is almost entirely absent. I am very pleased with its shape.

Oddly, that second ball made with my pattern (and with the same template), filled with poly-pellets, is only 2.3% heavier (144.8g vs 141.5g) and a few millimeters larger in circumference. I would have expected that the stretchier vinyl, producing a plumper, rounder ball,



My pattern using the stretchier and more flexible marine vinyl

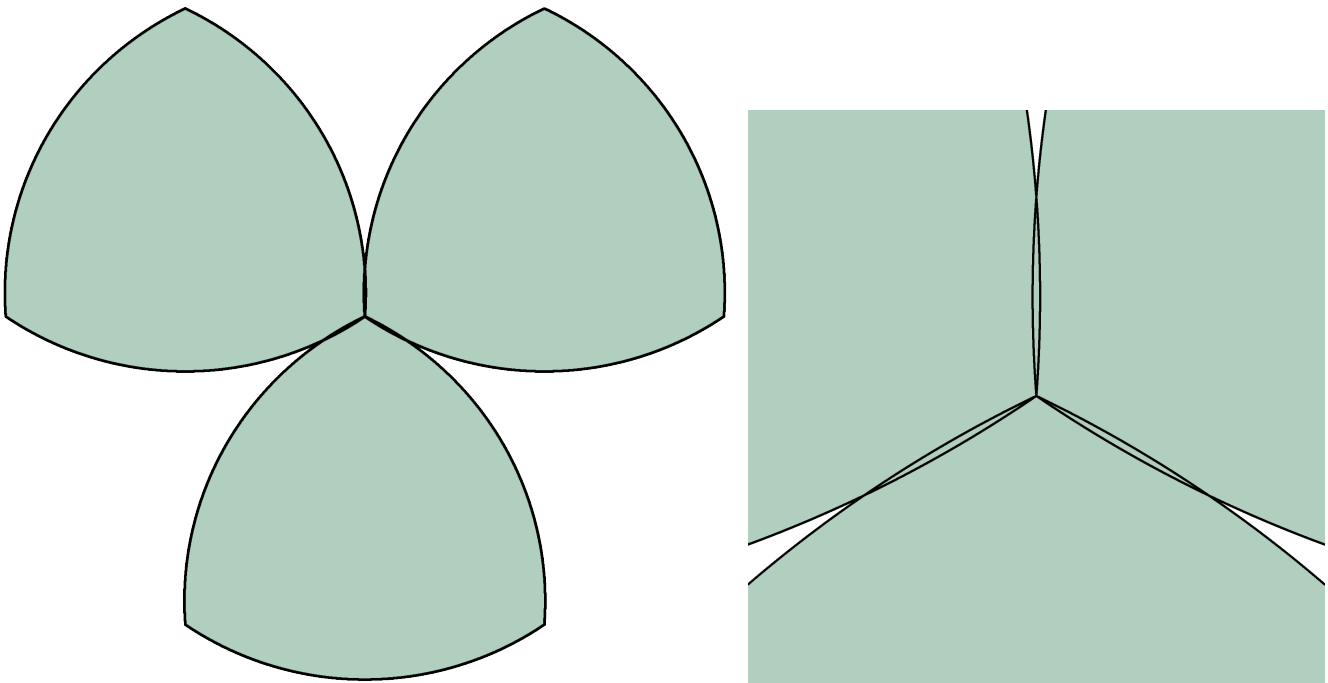
would have to stretch a lot more to produce its shape. Both balls were almost precisely 3 inches (~239mm circumference), which means my pattern sizing is excellent.

So I believe my spherical tetrahedron pattern to be definitely superior to the Reuleaux triangle, at least for approximating a sphere, and perfectly suitable for use with leather.

Bézier curve experiment

The illustrations below show the degree to which the pattern corners fail to fit together. My concern about my pattern was that when the curvature is too steep like this, though it is slight in this case, the resulting ball vertex can become too flat or even slightly inverted, and the seams can hump upward around the vertex, which I refer to as “shrugging their shoulders”. With stiff, non-stretch materials, the problem becomes more pronounced. This was a significant problem with the orange peel ball, but its curvature was much more over-steep than in this case. Still, I am striving for optimal spheres, and so I pay attention to very small imperfections.

However, in the case of the tetrahedron, I suspected that these characteristics of an over-steep curvature were actually a benefit. The asymmetrical nature of the polyhedron means that the vertex area needs to be flattened a little and widened so as to more closely match the shape of the face on the opposite side and produce a more balanced ball. And the steep angle of the panel faces from the vertices would likely counteract those characteristics anyway.



The corners of my pattern are wider than the 120° that would fit together, resulting in a small overlap.

Just to be sure that a corrected corner angle would not produce an unexpected effect, I created a Bézier curve pattern with 120° corners (illustrated below) and made a 3" ball with the stretchier marine vinyl. I chose that vinyl because I thought it would better show if the corrected corners produce sharper, more protruding vertices on the ball when the ball is stretched out with filler. The other vinyl is too stiff and puckers too much to produce a very smooth ball.

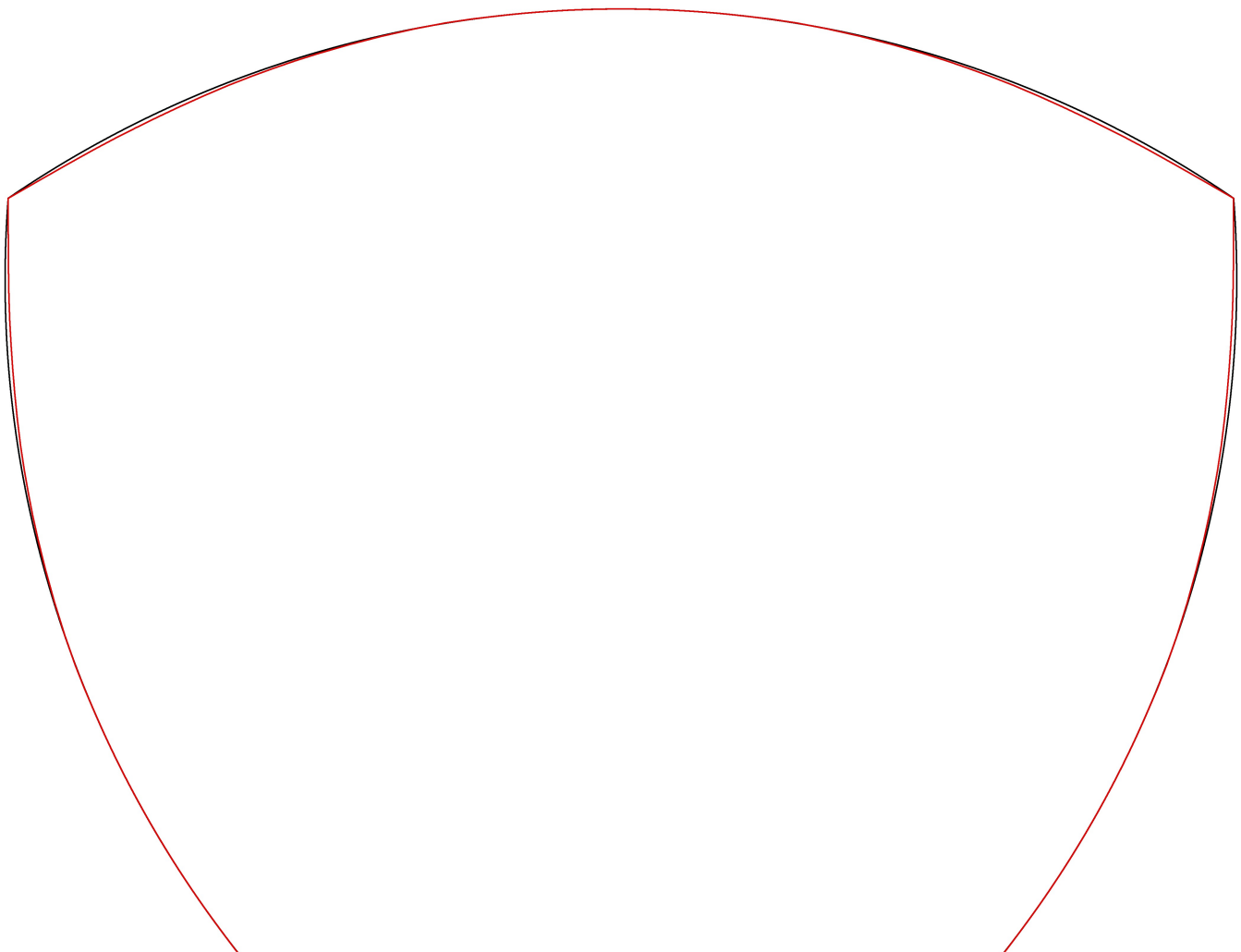
Though the Bézier pattern was hardly different from the circular pattern, such minute alterations have in the past surprised me by the noticeable changes they produced in the ball shape.

But in this case there was no discernible difference between the two balls. They are difficult to compare, though, because the stretchy vinyl has a bias effect and so the ball is a little lopsided and its faces and vertices are not uniform.

A few days later I made a pair of balls with my design testing fabric (thin, moderately stiff, tightly woven, non-stretch) and they were also identical as far as I could tell, and also lopsided. I aligned the fabric grain the same way on both balls to be sure that did not influence the comparison (I had not done that with the vinyl balls).

My instinct is to stick to the circular curve for the reasons I stated earlier, so I will leave the pattern unchanged unless a future experiment demonstrates that a different curve works better. This issue has nagged me in a small way for a long time and I am glad to have it mostly settled.

Below is an illustration of my Bézier curve (red) compared to the circular curve (black). The widest variance between the two curves in a 3" pattern is 0.25mm. To understand how I design my Bézier curves, read the "[Curve shape: Circular arcs versus Bézier curves](#)" topic in Chapter 5's "Curved-Edge Faces" section.



Bézier curve (producing a 120° angle at the corners) in red compared to the circular curve in black.